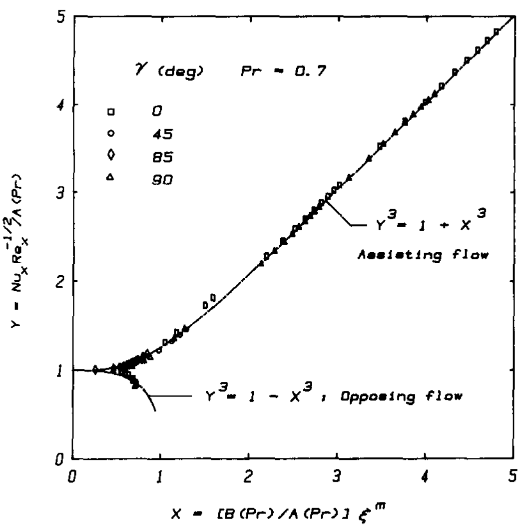


FIG. 3. A comparison between predicted and correlated local Nusselt numbers for vertical, inclined and horizontal plates.

results was also found to exist for the average Nusselt number with an exponent value of $n = 3$, as in the local Nusselt number.

## CONCLUSION

The local and average Nusselt numbers for laminar mixed convection flow adjacent to vertical, inclined and horizontal flat plates with uniform surface heat flux are presented for the entire mixed convection regime and for a wide range of Prandtl numbers. Simple correlation equations for Nusselt numbers are presented, which show an excellent agreement with the numerically predicted mixed convection values for both buoyancy assisting and opposing flows.

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# Large amplitude modulation of heat transfer from a circular cylinder

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## 1. INTRODUCTION

THE UNSTEADY heat transfer from a heated wire is encountered in various applications including hot-wire or film anemometry and electronic cooling. The time lag between the heat transfer and the relative velocity of the fluid to the wire is now well recognized. Using the Oseen approximation, Davies [1] analyzed the heat transfer from a constant-temperature circular cylinder in a cross-flow which has a small, sinusoidally fluctuating velocity superimposed on the mean velocity. The Reynolds number corresponding to the mean flow considered by him was smaller than one. Davies found that there is always a phase lag between the fluctuating velocity and the fluctuating heat transfer unless when the Reynolds number approaches zero. Apelt and Ledwich [2] studied the same problem for flows of Reynolds numbers in the range 1–40. They found numerically that the phase lag becomes more pronounced as the frequency of cylinder oscillation increases. Tseng and Lin [3] showed, by use of an asymptotic solution, that the phase lag persists in flows of Reynolds number of a few hundred. They also found the existence of an optimal frequency for the maximum heat transfer enhancement in a cross-flow of a given mean flow Reynolds number and a given small amplitude of cylinder fluctuation. Their asymptotic solution was later applied to construct the theory of a heat sensing velocimeter [4]. In the present investigation, we are concerned with the unsteady heat transfer from a heated cylinder oscillating with a large

## NOMENCLATURE

- $d$ diameter of cylinder
- $D$ distance between the wall and the cylinder axis normalized by $d$
- $g_\alpha$ gravitational acceleration component in the $\alpha$-direction
- $N$ half the horizontal computational domain normalized by $d$
- $p$ dimensionless pressure, $P/(\mu U_m/d)$
- $r$ radial distance from cylinder axis normalized by $d$
- $t$ dimensionless time, $t_1/(d^2/\nu)$
- $V$ dimensionless cylinder velocity
- $U_m$ average cylinder velocity
- $v_\alpha$ dimensionless velocity, $V_\alpha/U_m$
- $x, y$ dimensionless Cartesian coordinates, $X/d$, $Y/d$.

Greek symbols

- $\beta$ thermal expansion coefficient
- $\mu$ dynamic viscosity
- $\nu$ kinematic viscosity
- $\Delta T$ maximum temperature difference
- $\rho$ density
- $\omega_1$ dimensional frequency
- $\omega$ dimensionless frequency, $\omega_1/(\nu/d^2)$
- $\phi_j$ velocity or temperature basis function
- $\pi_{\alpha\beta}$ Cartesian stress tensor
- $\psi_j$ pressure basis function
- $\theta$ dimensionless temperature
- $\chi$ thermal diffusivity
- $\zeta$ polar angle measured counter clockwise from $+x$-axis.

Dimensionless groups

- $(Gr)_\alpha$ component of Grashof number vector, $\beta\Delta T d^3 g_\alpha/\nu^2$
- $Nu$ local Nusselt number, $-(\partial\theta/\partial r)_{r=1/2}$
- $\overline{Nu}$ average Nusselt number
- $Pr$ Prandtl number, $\nu/\chi$
- $Re$ Reynolds number, $U_m d/\nu$
- $S_t$ Strouhal number, $\omega_1 d/U_m$.

Subscripts

- $\alpha$ $x$ or $y$ Cartesian component
- $x, y$ $x$, $y$ components
- $j$ $j$th node of the finite element.

amplitude in a cross-flow. In addition to the usual phase lag, we found numerically, a hitherto unreported phenomenon of nonlinear amplitude modulation of the heat transfer response over a period which is one order of magnitude larger than the period of the velocity fluctuation. The numerical method used is a finite element Galerkin method. The detailed description of this method is available [5, 6] and will not be elaborated here.

## 2. NUMERICAL METHOD

Consider the unsteady two-dimensional heat transfer from a circular cylinder in a Newtonian fluid. The governing dimensionless equations with the Boussinesq approximation are

$$\begin{aligned} v_{\alpha,\alpha} &= 0 \\ v_{\alpha,t} + Re v_\beta v_{\alpha,\beta} + \theta (Gr)_\alpha / Re - \pi_{\alpha\beta,\beta} &= 0 \\ \theta_{,t} + Re v_\alpha \theta_{,\alpha} - \theta_{,\alpha\alpha}/Pr &= 0 \end{aligned} \quad (1)$$

where $v_\alpha$ is the velocity component normalized with the maximum velocity, $\alpha$ and $\beta$ stand for the $x$ or $y$ Cartesian coordinates divided by the cylinder diameter $d$, $\theta$ is the temperature normalized with the maximum temperature difference, $t$ is time normalized by $d^2/\nu$, $\nu$ being the kinematic viscosity, the subscripts following the comma, denote partial differentiations, and $\pi_{\alpha\beta}$ is the stress tensor. The parameters in (1) are $Re$, $(Gr)_\alpha$, and $Pr$ defined in the Nomenclature. $(Gr)_\alpha$ in a Cartesian coordinate system moving with the cylinder of velocity $-iU(t)$ is based on $g_\alpha = (\mathrm{d}U/\mathrm{d}t, g)$.

The initial condition is

$$\theta = v_\alpha = 0 \quad \text{at} \quad t = 0$$

and the boundary conditions with respect to a coordinate system attached to the cylinder moving at a velocity $-iU(t)$ are

$$\begin{aligned} &v_y = \theta = 0 \quad \text{and} \quad v_x = U(t) && \text{at} \quad x = -N \\ &v_{x,x} = v_y = \theta_{,x} = 0 && \text{at} \quad x = N \\ &v_y = v_{x,y} = \theta_{,y} = 0 && \text{at} \quad y = 0 \\ &v_y = \theta = 0 \quad \text{and} \quad v_x = U(t) && \text{at} \quad y = D \\ &v_x = v_y = 0 \quad \text{and} \quad \theta = H(t) && \text{at} \quad r = 1/2 \end{aligned}$$

where $H(t)$ is the Heavyside unit step function, $r$ is the radial distance measured in the unit of the cylinder diameter, and $2N$ and $D$ represent respectively the horizontal and vertical sides of the rectangular computation domain. Successively larger values of $N$ and $D$ will be chosen to meet the required numerical accuracy. The above boundary conditions imply that the flow is symmetric with respect to the $x$-axis. Hence the results to be presented are for flows before the asymmetric vortex shedding, and for the case of negligible asymmetric natural convection.

The solution of (1) is expanded in terms of the basis functions $\theta_j$ and $\psi_j$ [5, 6]

$$\begin{aligned} (v_\alpha, \theta) &= \sum_{j=1}^{n} [v_{\alpha,j}(t), \theta_j(t)]\phi_j(x, y) \\ p &= \sum_{j=1}^{m} P_j(t)\psi_j(x, y) \end{aligned} \quad (2)$$

where $p$ is pressure normalized with $\mu U_m/d$, $\mu$ being the dynamic viscosity, and $n$ and $m$ are respectively the total number of velocity or temperature and pressure nodes in the finite element method. The Galerkin method is applied to reduce equations (1) with (2) to a system of ordinary differential equations in the amplitude functions $v_{\alpha j}(t)$, $\theta_j(t)$ and $P_j(t)$. The system is then solved with a predictor–corrector method [5, 6].

## 3. RESULTS AND DISCUSSIONS

First the method is applied to the case of impulsively started uniform motion of a cylinder, i.e. $U(t) = H(t)$. The local Nusselt number, $Nu$, is computed at each time step from the temperature gradient at the cylinder surface.

$$Nu(\zeta, t) = -\frac{\partial}{\partial r}\theta(r, \zeta, t)\Big|_{r=0.5}$$

where $\zeta$ is the polar angle measured from the $x$-axis, and $r$ is the radial distance measured from the cylinder axis. The results of the local Nusselt number of $\zeta = \pi/4$ are given in Fig. 1 together with other known results. Comparisons are very good except with that of Jain and Goel [7] at large times. The steady-state local Nusselt number variation along the cylinder surface is also obtained for $Re = 200$ and $Pr = 0.7$.

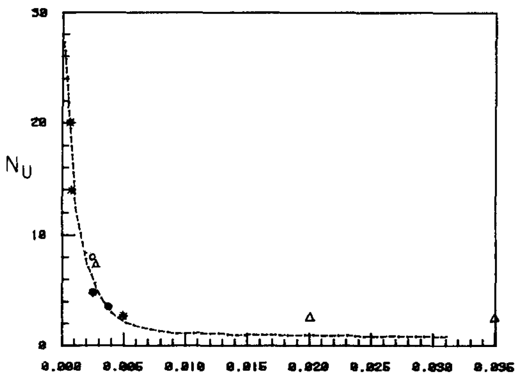


FIG. 1. The local Nusselt number at $Re = 200$, $Pr = 0.7$, $\zeta = \pi/4$: --- present study; ○ Sano [8]; * Tseng and Lin [3]; △ Jain and Goel [7].

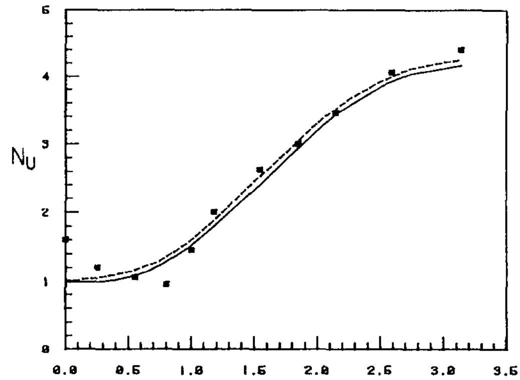


FIG. 2. The local Nusselt number at $Pr = 0.7$, $\zeta = 0$: —— present study ($Re = 20$); --- Dennis *et al.* ($Re = 20$) [10]; □ Eckert and Soehngen [9].

The results are given in Fig. 2 together with the experimental results of Eckert and Soehngen [9] and the numerical results of Dennis *et al.* [10]. Although the numerical results obtained with two different methods agree well in Fig. 2, both do not reproduce Eckert's local minimum of $Nu$. The discrepancy is probably due to the difference in the boundary conditions encountered in the experiments and those encountered in numerical computations. Other than this discrepancy, the agreements are good.

Having applied the method successively to the case which has known experimental and numerical results, we now applied the method to produce some new results. Consider a circular cylinder of constant surface temperature in a cross flow of constant velocity $U_m$. Initially the temperature and velocity field about the cylinder are steady. Then, at a certain time the cylinder is made to oscillate. Thus, the free-stream velocity, nondimensionalized with $U_m$, relative to the cylinder, is

$$U(t) = 1+\delta \sin \omega t$$

where $\omega$ is frequency of oscillation nondimensionalized with $v/d^2$, and $\delta$ is the amplitude of oscillation normalized with $d$.

The numerical results to be presented are all for the value of $\delta = 0.5$. The average Nusselt number, that is the local Nusselt number integrated over the entire cylinder surface, has been computed as a function of time for a given $Re = 20$ and two different values of frequencies. The Reynolds number is based on the average velocity. The results are given in Fig. 3 together with the commonly used quasi-steady correlation [11]

$$\overline{Nu}_s = C[U(t)d/v]^n$$

where for $4 < Re < 40$, $C = 0.821$ and $n = 0.385$. Note the phase lag between the quasi-steady heat transfer and the genuine unsteady heat transfer of $0.3\pi$ in Fig. 3. The phase lag of local Nusselt number is even more dramatic. The phase lag at the rear 'stagnation point' was found to be larger than that at $\zeta = \pi/4$ by a value of $0.4\pi$. The phase lag has also been observed for much smaller frequencies. Dennis *et al.* [10], using Oseen type approximation, estimated that the minimum value of the Strouhal number of the impressed fluctuations at which the time lag becomes important is of order $0.1\ Re \cdot Pr$, where the Strouhal number is related to $\omega$ by

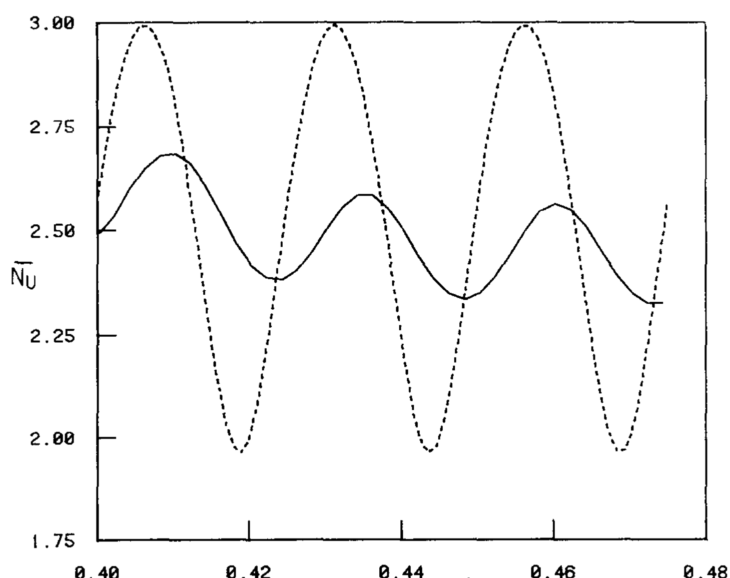


FIG. 3. Responses of average Nusselt number to periodic fluctuation in velocity about a mean flow at $Re = 20$, $Pr = 0.7$, $\omega = 80\pi$: —— computed responses; --- quasi-steady response.

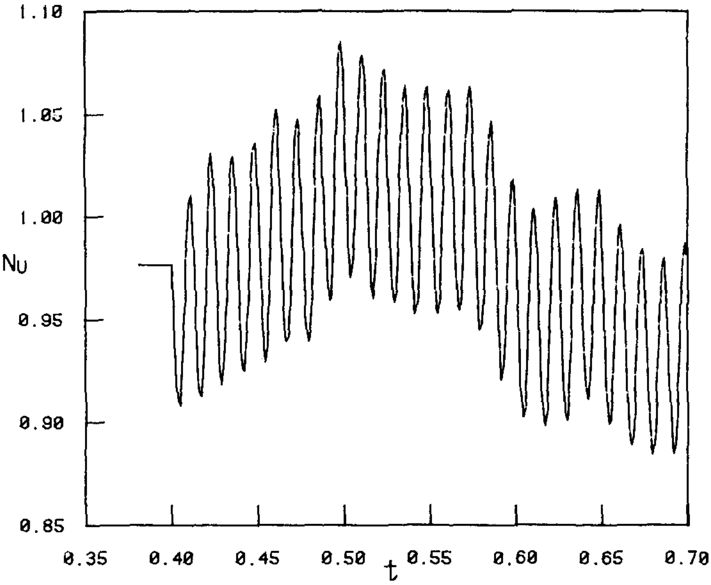


FIG. 4. Amplitude modulation of local heat transfer coefficient at $\zeta = 0$, $Re = 20$, $Pr = 0.7$, $\omega = 160\pi$.

$$\omega = S_t Re.$$

Dennis *et al.* also found that the phase lag approaches $\pi/2$ as $S_t \to \infty$. The results obtained by Apelt and Ledwich [2] for the frequency corresponding to a Strouhal number of 0.87 showed a phase lag of $0.228\pi$ at $Re = 20$. Comparing these known results with the present results, we conclude that the time lag appears to increase with the frequency for all finite $Re$.

Figure 4 shows the time variation of the local Nusselt number at the rear stagnation point. Note that even after 23 cycles, the periodic stationary response has not yet been reached. Moreover, the response curve seems to modulate over a period which is one order of magnitude larger than the period of the cylinder fluctuation. This nonlinear modulation cannot be predicted from the quasi-steady correlation.

When our results for $Re = 20$ are applied to air flow in the 1–10 m $s^{-1}$ range, the relevant cylinder diameter is smaller than 1 mm. The dimensional frequencies corresponding to $\omega = 80\pi$ and $160\pi$ used in our demonstration require high dimensional frequencies of order 10 kHz. No experiments in this range of parameters are known to the writers.

All computations was done on the IBM 4341 and VPS 32 with double precision. When the horizontal extent of the computational domain was doubled by increasing $N$ from 10 to 20 with $D = 5$, a change of less than 5% in the numerical results was found. The change becomes less than 1% when $N$ was further increased to 40. When $D$ was increased from 5 to 10 with $N = 20$, a change of less than 1% was found. $N = 20$ and $D = 5$ were used for the results reported in this work. For the sufficient spatial resolution, we halved the finite-element size near the cylinder successively until the two successive results differ from each other by less than 1%. The time steps were chosen adaptively with the requirement that the truncation error is below a given value of $10^{-4}$ at each time step [5].

## 4. CONCLUSION

A computer code based on the Galerkin finite element method has been developed for solving the problem of two-dimensional unsteady heat transfer. The code can be applied to any geometry, although the code is applied here to the case of a circular cylinder between two parallel plates. This code will be made available to interested readers. The time lag in $Nu$ and $\overline{Nu}$ and their nonlinear modulation demonstrated in this work should be taken into account in any application involving unsteady heat transfer.

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